Climate Change and Optimal Rotation in a Flammable Forest

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Running head: Fire and forest rotation

Symbols Used

-	alpha
*	delta
8	lambda
D	rho
(	gamma

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Abstract

This paper builds a Faustmann-based model to investigate the effects of increased climate-induced fire risk on the optimal rotation period in a commercial forest. Simulations using species of trees prevalent in North American forests indicate that both the commercial and socially optimal rotation ages decline as the risk increases. This occurs despite the fact that the inclusion of carbon sequestration benefits in society's maximand means that the socially optimal rotation length exceeds the length that is commercially profitable.

The increased fire risk as the climate warms also has important implications for the ability of forests to act as absorbers of carbon. The arguments of the 'Umbrella Group' of countries who desire to use their forests' carbon-absorbing ability to offset their need for fossil fuel emission reductions will have increasingly less force as the climate warms. Because the heightened fire risk dramatically reduces the ability of living forests to act as carbon sinks, alternative proposals for storing carbon by 'pickling' wood in cold lakes look increasingly attractive.

Keywords: atmospheric carbon, boreal forests, carbon storage, climate change, Faustmann, forest fires, forests, global warming

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1. Introduction

From December 1 to 10, 1997, ministers and other high-level officials from 160 countries met in Kyoto, Japan, for the Third Conference of Parties to the United Nations. Under the Kyoto Protocol that resulted from this conference, industrialized countries must reduce their collective emissions of greenhouse gases by 5.2 percent below 1990 levels by the period 2008 to 2012. While this is a modest target in that carbon dioxide levels have increased about thirty percent since the industrial revolution, its accomplishment by means of reductions in the use of fossil fuels appears more and more unlikely. Because forests and other plant matter absorb carbon dioxide, an alternative to reducing anthropogenic emissions might be to increase the biomass of global forests, the use of new carbon sinks being specifically permitted under Article 3 of the Kyoto Protocol. This possibility has been the major stumbling block to agreement on the implementation of the Kyoto Accords, the so-called Umbrella Group of countries arguing that their forests allow them to forego fossil fuel emission reductions.[1] Proposals for creating new carbon sinks include reducing the rate of deforestation, replanting fast-growing forests (van Kooten (1999), or switching from fossil to biofuels (Sedjo and Solomon (1989); Dixon *et al.* (1993); Binkley *et al., (*1997)). Economic research has so far focussed on the relative costs of sequestering carbon with the different options (Moulton and Richards (1990), Nordhaus, (1991)).

A problem with this possible solution to global warming is that forests themselves periodically emit large amounts of carbon as the consequence of fires. For example, from 1850 to 1980, an estimated 90 to 120 billion metric tons of carbon dioxide were released into the atmosphere from

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[1] The Umbrella Group comprises Australia, Canada, Japan, New Zealand, and the United States.

tropical forest fires, compared to the 165 billion metric tons added to the atmosphere through the burning of coal, oil, and gas. Wildfires are now a dominant ecological disturbance, and their size and frequency appear to be increasing.[2] Today the burning of tropical forests alone contributes about thirty percent of the total carbon emissions[3]. There is much recent research on carbon uptake and emissions within the forest sector, and on the complex interactions between warming, carbon absorption and forest fires. Important positive feedback effects appear to be in operation. It is widely expected that warming with increase the likelihood of forest fires, which will then increase the forest carbon emissions, as indicated by a number of simulation studies (Kurtz and Apps (1994), (1995); Peng and Apps (1999) and earlier studies reviewed by Harrington (1987)). Many of these studies also indicate that the greatest warming engendered by increasing CO_2 will occur at higher latitudes (45 to 65 degrees N) with the most marked effects within the continental interiors (Stocks, Lee and Martell, (1996)). The doubled CO_2 experiment by Mitchell (1983), for example, produced differences of between three and ten degrees in the mean winter surface temperature for much of the land surface area of the boreal zone, and other studies have indicated that there may be significant warming and drying in the summer months in the same region, increasing the fire frequency. However, a possible long-term negative feedback effect may be more rapid forest growth due to the warming itself, absorbing more carbon and eventually slowing the warming.

These possible ecological interactions are under intense study by forest biologists and ecologists, but there has been relatively little research on the effects of human adaptation to warming within the forest sector. This adaptation may include increased fire suppression expenditures or changes in harvest rotation age on the part of forest managers. While the former is possible, it is not likely in the face of increasing budget pressure on the part of most governments, who are the *de facto*

[2] One of the largest fires in the recent past destroyed more than twelve million acres of boreal forest in Russia and the People's Republic of China during May, 1987.

[3] Smith *et al.* (1993) estimate annual carbon emissions from fossil fuel burning of about 6 billion tonnes. Deforestation implicitly adds another 2 billion tonnes through reduced absorption.

managers of many of the world's forests. This paper instead focusses on the rotation length decision, building on the standard Faustmann model of optimal rotation.

If an increased forest biomass slows global warming, it creates a positive externality for the rest of society, and a few papers within the resource economics literature have considered this in a general way, building on the original Hartman model (Hartman, (1976); Calish, Fight and Teeguarden, (1978)). Haynes, Alig and Moore (1994) tried to estimate this effect using simulations for U.S. forest species. Englin and Calloway (1993) and Martin (1998) compare commercially optimal rotations and the socially optimal rotation lengths resulting when carbon sequestration benefits are included.

Within the forestry literature, a number of papers have studied the effect of a parametrically increased fire risk on rotation age. Martell (1980) used a simple model to determine the optimum rotation of a forest stand that is threatened by fire, and similar models were used by Reed (1984), Martell (1994) and Englin, Boxall and Hauer (2000). In these models, an increase in the risk of fire acts in a similar way to an increase in the discount rate, tending to reduce the (constant) planned age of rotation. However, these models did not allow the fire risk to change over time as the climate warms. Such inter-temporal effects are similar to the effects of time-changing forest prices or costs studied by McConnell, Daberkow and Hardie (1983) and Newman, Gilbert and Hyde (1985). This paper essentially generalizes the Reed model by allowing the fire risk to change over time. We will show that increased risk results in a reduction in both the commercially and socially optimal forest rotation periods.

The plan of the paper is as follows. The subsequent section outlines the model of optimal rotation for a commercial forest in the presence of increasing fire risk. This model reduces to the Reed model when risk is constant, and to the Newman, Gilbert and Hyde model when there is no fire risk but an exogenous trend in forest rents. A simulation of this model with three forest species is performed in section three, illustrating the changes in the commercially optimal rotation periods for approximately two hundred and fifty years into the future. The next section adds carbon sequestration benefits to the model, and shows that the socially optimal rotation length also declines with climate warming. The final section discusses the implications of climate warming for the possibility of using forests as carbon sinks.

2. A generalized Faustmann model with an increasing risk of fire

We assume a forest manager determining the optimal sequence of rotation ages on a given forest stand. The stock of trees on this stand is subject to random disturbances, the major ones being fire and insect infestation. In conformity with most fire risk models and with empirical studies of fire risk, we assume that a Poisson process governs the number of years until a fire, so that the time between forest fires follows the exponential distribution function $\left(1 - e^{-\lambda x}\right)$ which has a probability density function $\lambda e^{-\lambda x}$. λ is the conditional risk given survival to date, which in this case is the inverse of the mean time between fires. Thus if λ=1/60, there is on average 60 years between fires. Unlike previous models, we will specify λ as a function of time.

In the presence of fire risk, the value of one rotation, terminated by fire or harvest, is the random variable B(x,a), which is a function of x, the random waiting time between fires or harvest, and the planned rotation age, a. Given revenue at harvest R(a) and a planting cost c, (compounded to the end of the rotation period at discount rate r), the net return at age x is:

$$B(x,a) = \begin{cases} -ce^{rx} & if \quad x < a, \\ R(a) - ce^{ra} & if \quad x = a. \end{cases}$$

Each rotation period ends either at the planned harvest period or with a fire, in which case x < a, and no revenue is obtained. For simplicity, no additional site preparation cost is assumed in the case of replanting after a fire. With the given distribution for the probability of a fire, the chance that the stand is destroyed before the planned harvest time a is $1 - e^{-\lambda a}$, and the probability of harvest is $e^{-\lambda a}$.

Denoting by E the expectation operator, the expected present value of the series of rotations of harvests and burns is the summation

$$(1)\qquad PV = \sum_{n=1}^{\infty} E\left(e^{-rt_n}\pi_n\right) = \sum_{n=1}^{\infty} E\left(e^{-rt_{n-1}}\right)E\left(e^{-rx}\pi_n\right).$$

In this equation, t_n is the time at the end of the nth rotation. As $t_n = t_{n-1} + x$, (with $x \leq a$) the expression can be factored as shown. However, the waiting times between harvests are not necessarily independent, and the planned harvest interval is not necessarily constant when λ varies with time, so that the last term is not constant and able to be factored out of the summation as in Reed (1984) or Englin, Boxall and Hauer (2000). Let us designate by δ_n the expected discount factor $E(e^{-rt_n})$, and define $\delta_x(a,\lambda) = E(e^{-rx})$ as the expected discount factor over one rotation. Although we cannot evaluate δ_n explicitly, with our given distribution

$$(2)\qquad \delta_x(a,\lambda) = E(e^{-rx}) = \frac{\lambda + re^{-(r+\lambda)a}}{r+\lambda}.$$

with $d\delta_x/d\lambda > 0$ and $d\delta_x/da < 0$. The expected discount factor then evolves over time as $\delta_n = \delta_{n-1} * \delta_x$. The expected rents over one rotation are defined as

$$(3)\quad V_n(a_n,\lambda) = E(e^{-rx}\pi_n) = R(a)e^{-(r+\lambda)a} - c[e^{-\lambda a} + \int_0^a \lambda e^{-\lambda x}dx] = R(a)e^{-(r+\lambda)a} - c,$$

so that an increase in the risk has the same effect on the expected rents over one rotation as an increase in the discount rate. We shall model the effects of climate change on the forest as an increase over time in the conditional risk, λ. Specifically, we model λ, the risk of fire in the current rotation, as a declining function $\lambda(\delta_{n-1})$ of the discount factor in the previous rotation, so that λ increases as δ declines over time.[4] Including the constraint, to maximize expected forest rents the forest manager then chooses a sequence of planned rotation ages a_n to maximize the Lagrangian:

[4] For the empirical simulation, this is specified as $\lambda = \lambda_0 - \alpha \ln(\delta_{n-1})$ so that, if δ were non-stochastic and equal to $e^{-rt(n-1)}$, λ would be a linear function of time.

(4) $$PV = \sum_{n=1}^{\infty} \rho_{n-1} V_n(a_n,\lambda) + \gamma_n\left[\rho_n - \rho_{n-1}\rho_x(a_n,\lambda)\right],$$

where γ_n is the shadow price associated with changes in the expected discount factor.

The first-order conditions for this maximization are:

(5) $$\frac{dPV}{da_n} = \rho_{n-1}\left(\frac{dV_n}{da} - \gamma_n\frac{d\rho_x}{da}\right) = 0$$
$$\frac{dPV}{d\rho_{n-1}} = V_n + \rho_{n-1}\frac{dV_n}{d\rho_{n-1}} + \gamma_{n-1} - \gamma_n\left(\rho_x + \rho_{n-1}\frac{d\rho_x}{d\rho_{n-1}}\right).$$

In these equations the derivatives $dV/d(\rho_{n-1})$ and $d\rho_x/d(\rho_{n-1})$ follow from the implicit functions $V(\lambda(\rho_{n-1}))$ and $\rho_x(\lambda(\rho_{n-1}))$. Combining them, the rotation age a_n is determined implicitly from the following recurrence relation:

(6) $$\rho_{n-1}\left[\frac{\partial dV}{\partial\rho_{n-1}} - \left(\frac{\partial V/\partial a}{\partial\rho_x/\partial a}\right)\frac{\partial\rho_x}{\partial\rho_{n-1}}\right] + \left(\frac{\partial V/\partial a_{n-1}}{\partial\rho_x/\partial a_{n-1}} - \rho_x\frac{\partial V/\partial a_n}{\partial\rho_x/\partial a_n}\right) + V = 0.$$

One can simplify this by using the fact that increasing the planned rotation age indirectly increases the expected discount factor between rotations, ρ_x. Substituting $dV/d\rho_x$ for $(\partial V/\partial a)/(\partial\rho_x/\partial a)$ etc. we can rewrite (6) in terms of the capital gain or loss from waiting and allowing ρ_{n-1} to increase over time, (V plus the first term in the square bracket) and the returns from indirectly changing the expected discount factor:

(6') $$\rho_{n-1}\left[\frac{\partial dV}{\partial\rho_{n-1}} - \frac{dV}{d\rho_x}\frac{\partial\rho_x}{\partial\rho_{n-1}}\right] + \left(\frac{dV}{d\rho_x}\Big|_{a_{n-1}} - \rho_x\frac{dV}{d\rho_x}\Big|_{a_n}\right) + V = 0.$$

The first bracketed term of this relationship results from the change in λ over time, while the last three terms alone determine the (stationary) rotation age in the Reed model with a constant fire risk. To relate it to the more familiar Faustmann formula we will show that our relationship encompasses both the Reed case of constant λ and the case employed by McConnel *et al.* and Newman *et al.*, in which returns are non-stochastic but there is a change in profits over time.

The Constant λ Case

If λ is constant, $dV/d\rho_{n-1}$ and $d\rho_x/d\rho_{n-1}$ are zero, so a_n is determined in equation (6') at the point where V plus the last term in brackets equals zero. In this case, the waiting times between harvests are independent, and the expected discount factor and expected value per rotation are both constant, so a_n will also be constant. Equation (6') above then becomes simply $V + [1 - \rho_x](dV/d\rho_x) = 0$. This is the result of maximizing PV with ρ_x inter-temporally constant,[5] or

$$(7) \qquad PV = \sum_{n=1}^{\infty}\left[\prod_{i=1}^{n-1}\rho_x\right]V = \frac{V}{1-\rho_x}$$

Substituting for V, etc., the optimum condition in this case is equivalent to the standard Faustmann formula with a risk-adjusted discount rate, becoming

$$(8) \qquad R'_a + (r+\lambda)R(a) + rPV = 0,$$

which is equivalent to Reed's equation (16), p. 184. With λ constant, it is easy to show that as long as R"(a) < 0 a parametric increase in the fire risk will shorten the (inter-temporally constant) planned rotation age. Totally differentiating (8) above and noting that dPV/da = 0,

[5] ρ_x is now a function only of a_n, so a_n and ρ_x are equivalent choice variables in maximizing PV.

$$(9) \qquad \frac{da}{d\lambda} = \frac{R(a)}{R''_a - (r+\lambda)R'_a} < 0.$$

The Model without Risk but π Evolving over Time

If the returns are nonrandom, x then equals a, the planned rotation age. As well, the expected and actual discount rates are equal, with $E(e^{-rt_n}) = e^{-rt_n}$ and $\rho_x = e^{-ra}$. Although time affects rents in the model only through the trend in λ, (and λ is zero when rents are non-stochastic), for comparison with previous papers on trending forest prices let us simply allow harvest revenues to depend on the date of the previous rotation, so that the rents at the end of the nth rotation period equal $\pi(a_n, t_{n-1}) = R(a_n, t_{n-1}) - c_p e^{ra}$ and the beginning of period net return is $V = e^{-ra}\pi(a_n, t_{n-1}) = e^{-ra}R(a_n, t_{n-1}) - c_p$. Now, $dV/d\rho_x = V_a/\rho_{xa} = -(\pi_a - r\pi)/r$ and $dV/d\rho_{n-1} = -e^{-ra}\pi_{t_{n-1}}/(r\,\rho_{n-1})$.[6] Substituting into equation (6) with $d\rho_x/d\rho_{n-1} = 0$ and simplifying, a_n is determined from the following implicit difference equation:

$$(10) \qquad \pi_{t_{n-1}} - \pi_{a_n} + \left(\pi_{a_{n-1}} - r\pi_{n-1}\right)e^{ra_n} = 0.$$

This says that a_n is chosen at the point where π_{a_n}, the marginal return from an additional year's growth, equals the exogenous increase in profits over time plus the compounded net return in the previous rotation. This relation is equivalent to the result in Newman, Gilbert and Hyde (1985). As well, when $\pi_t = 0$ and both the rotation age and π_a are constant, (10) reduces to the normal Faustmann formula $\pi_a(1 - e^{-ra}) - r\pi = 0$. When profits change over time it is not possible to determine the shape of the time path for a_n, from rotation to rotation, without assuming particular functional forms in solving this nonlinear difference equation. Finding the path for the rotation age in the general model thus must be found by simulation, which is done in the next section.

[6] $d\rho_{n-1}/dt_{n-1} = -r\,\rho_{n-1}$, so $dt_{n-1}/d\rho_{n-1} = -1/(r\,\rho_{n-1})$

3. Simulation of the model of section two with parameters from Canada's boreal and subarctic forests

In this section, the model is simulated with the growth rates for the tree species Douglas fir, white spruce and jack pine. Douglas fir is the dominant species on the west coasts of Canada and the U.S. White spruce is prevalent in the Western Canada interior boreal and subarctic forests, where the predicted warming is greatest, while jack pine is typical in Northern Quebec and Ontario forests.

Forests occupy about 400 million hectares in Canada, or roughly half of the land area, and represent approximately 10% of global forests. The boreal zone is about 290 million hectares. Most of Canada's forest land is publicly owned. Only 6 per cent belongs to private interests, while 71 per cent comes under provincial jurisdiction and 23 per cent is the responsibility of the federal and territorial governments. Of this forest land, 56 per cent (236.7 million hectares) is classified as 'commercial,' being leased by the Crown to private forest companies, but only 119 million hectares of this commercial forest is currently used for timber production, because of the high access and transport costs in the Canadian north. Canadian commercial forests are still managed indirectly by the Crown in that many decisions are made by Government officials, not strictly on commercial grounds. Since 1992, sustainable development, however defined, has been the ostensibly guiding principle of the National Forest Strategy, and the provinces of British Columbia, Saskatchewan and Quebec have all established legislation based on sustainability principles. Thus we cannot predict the Canadian forest industry's response to climate change from a modified Faustmann formula. Despite this, it is useful to characterize the nature of a manager's response based solely on commercial considerations, because this is an important factor in forest decisions.

As elsewhere, Canadian forests are subject to increasing rates of disturbance due to fire and pests. Over the past ten years, an average 9,600 wildfires have burned 2.9 million hectares annually (0.6% of the total forested area), and the area burned has increased substantially since 1960. For example, the number of fires recorded between 1960 and 1995 was 60% higher than the total for 1920–1960. From 1920-1969 the average area disturbed by fire and pests was 1.7 million hectares/yr, and this doubled to 3.6 million ha/yr in the 1970-1989 period. However, the 1920-1969 fire rate was actually unusually low in Canada, and consequently the average age of the boreal forest increased from 60.9 years in 1920 to 82.5 years in 1970, before declining to 76.4 in 1989 (Kurtz and Apps (1999)). Figure 1, from the Canadian Forest Service, shows the annual variability in Canadian forest fires. The increased disturbance rate is the reason for a recent dramatic reduction in net carbon absorption within Canadian forests.

The global warming trend has also been evident in the Canadian north. Summer temperatures in the Canadian boreal forest have increased an average one degree Celsius over the last century; .5" in the east, 1.4" in the western boreal, and 1.7" in the northern Mackenzie river district, and there has been a lot of research on the relationship between climate change and increased wildfires. To predict the forest burn rate, foresters use a weighted average of temperature, moisture and other data called the fire weather index (Hirsch, (1993); Stocks (1993), Stocks and Kaufmann, (1997). Using three commonly used general circulation climate models, Flannigan and Van Wagner, (1991) predicted that a twice CO_2 climate would increase the seasonal severity of Canadian forest fires by 46 percent.

To simulate the model in the previous section, we first specify harvest revenue R(a) as the stumpage price p times tree harvest volume v(a). We then require estimates of growth functions for the tree species, as well as the real discount rate, r, the cost/ price ratio c/p, the initial burn rate, 8_0, and estimate of its time trend. The real discount rate was set at 2 %, at the middle of the 1% - 3% historical range for the real government borrowing rate (Lind (1990)). The stumpage price for

Douglas fir logs has been approximately p = \$1200 per 1000 cu. ft. (or \$42.40 per m^3) in the last decade, and a reasonable Douglas fir planting and site preparation cost value of c = \$300 per acre or \$741.30 per hectare was obtained via Martin(1998) from a U.S. Forest Service study of U.S. Pacific coast tree plantations (Moulton and Richards (1992)). Thus, the cost-price ratio for Douglas fir in metric units is approximately 17.5. While information was obtainable on spruce and pine stumpage prices, there was none on site preparation costs in the other regions. Consequently, the cost-price ratios for these species were set at levels that gave single-rotation expected pre-tax rates of return per hectare at the constant fire-risk rotation age that were approximately equal to the 39.1% return for Douglas fir. This occurred at c/p = 7 for jack pine and only c/p = .1 for white spruce[7].

The growth function parameters for the three typical tree species were obtained from various sources. The equation for Douglas fir was obtained from Martin(1998), who estimated Douglas fir volume by age for U.S. Pacific Northwest site index 160 based on data obtained from the U.S.D.A. Forest Service. The equation for northern Ontario jack pine was obtained from from Martell (1980) and the volume growth relationship for white spruce on the Aralia site of north-central British Columbia was derived from a paper by Craigdallie and Simmons (1982). These growth function equations, converted to cubic volume per hectare (m^3), are shown in Table 1 and illustrated in Figure 2.

For the simulation, the fire risk was specified as the function $8 = 8_0$ - " ln(D_{n-1}) of the expected discount factor in the previous rotation. This relationship implies that if D_{n-1} were non-stochastic and equal to $e^{-rt(n-1)}$, then 8 would equal 8_0 + "rt, a simple linear function of time. We however need the parameters 8_0 and ". The initial burn rate 8_0 was set at .014, the inverse of the seventy-year average disturbance cycle in the Canadian boreal forest in the early 1990s. Estimating the parameter ", representing the trend in the fire disturbance rate, is of course much more problematic. One possible source is the paper by Johnson and Larsen (1991), which undertakes an historical analysis of climate-related changes in fire frequency in Canada's Southern Rocky Mountains during the last 380 years.

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[7] The expected single-rotation rate of return on sales is V(a)/R(a) = [v(a) - c/p]$e^{-(r+8)a}$/(v(a)).

According to these forest scientists, the change from the warmer and drier climate prior to 1730 to the cooler and wetter one post-1730 increased the fire cycle from 50 to 90 years, or equivalently reduced the disturbance rate from .02 to .011 in this region. This would imply α equal to $\Delta\lambda/r\Delta t$ = $1.48*10^{-3}$, and a disturbance rate in 400 years of 2.5% instead of the current 1.4%. Alternatively, simply using the change in the overall age of the Canadian boreal forest in the 1970-95 period results in a higher α at $2.42*10^{-3}$ and a predicted $\lambda(400)$=3.6%, while a regression of the burn rates against time for the 1970-95 period gives $\alpha = 9.88*10^{-3}$ and a very high future disturbance rate of 6.9%.[8] From these three estimates, we have used an intermediate rate of $\alpha = 2.0*10^{-3}$ for the simulation, implying the future disturbance rate $\lambda(400)$ of approximately 3%.

To begin the simulation exercise, we first calculated the steady-state Faustmann rotation age for each species with a discount rate of r = 2 %, the cost/price ratio as calibrated, and λ equal to zero. At these base-case parameter values, the Faustmann optimal rotation ages were 51.4, 41.0 and 58.3 years for Douglas fir, jack pine and white spruce respectively, as shown in the first row of Table I.. Next, the model was simulated for a constant disturbance probability, with λ equal to its current rate of 1.4% and α zero (equation (8) of the previous section). These results are shown in the second row of the table. Within this constant-risk model the effects of parametric changes in the discount and disturbance rates were then investigated. As expected, increasing the real discount rate reduces the rotation age, as does doubling the risk from λ = .014 to .028, these effects shown in the last two rows of the table. To illustrate the equivalence of a constant fire risk to an increased rate of discount, the discount rate increase is to 3.4%, the sum of the original discount rate and the 1.4% fire risk.

Introducing a climate trend into the model no longer generates a fixed rotation age but a sequence of ages for different rotations. Equation (6.1) of the previous section is an implicit nonlinear function $f(a_i, a_{i-1}, \rho_{i-1}, \rho_{i-2}) = 0$ of a_i and $a_{i-1,}$ the ages of the current and previous rotations, and the

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[8] Data from 1918 to 1969 were not used for the estimation, despite its availability in Canada, because this period is regarded as one of unusually low disturbance.

lagged expected discount factors D_{i-1} and $D_{i-2.}$ The expected discount factor D_i also adjust from rotation to rotation according to $D_i = D_{i-1}$ times $D_x(a_i, D_{i-1})$, the expected discount factor over a single rotation. These two simultaneous equations in a_i and D_i were repeatedly solved using the numerical equation solver in the Mathcad programming language as functions of the lagged values a_{i-1}, D_{i-1}, and $D_{i-2.}$ The simulation was begun with D_0 equal to unity, a_1 equal to the constant risk rotation age above, and $D_1 = D_x(a_1, D_0)$, the expected first-iteration discount factor. Simulations were run for twenty rotations, and stopped when expected rents per rotation became zero, which occurred at less than 10 rotations, or between 250 and 300 years for each species. Figure 3 shows that the resulting rotation ages for each of the three forest species unambiguously decline (becoming zero when expected rents are zero). This pattern is not sensitive to parameter changes, such as changes in discount rates, planting costs and the rate of increase in the disturbance risk, within reasonable limits. It thus appears that increased risk acts in a similar way to a discount rate that increases over time, inducing a reduction in the profit-maximizing rotation period. The burn rate, 8, increased from 014 to .023 over the approximately 250 year simulations.

4. The socially optimal rotation length when carbon sequestration benefits are added

Adding the social carbon-absorbing benefits of standing timber to the maximand in equation (4) will change the optimal planned rotation age. The total carbon stored in a typical stand follows a similar pattern to wood volume, with the rate of carbon sequestration $\dot{\theta}(a)$ increasing and then declining as the trees age. This is illustrated by the Douglas fir carbon absorption function derived by Englin and Calloway (1993), shown in Figure 4 as tons of carbon stored per hectare per year. The maximum absorption rate according to this function is at 48.4 years, compared with as the commercially optimal 43.8 year rotation period for a Douglas fir stand given the current 1.4% fire risk. Because fire instantly releases the stored carbon back into the atmosphere, however, the relevant concept when fire risk is present is the expected present value of sequestered carbon. To derive this, we must weight the values of the various transfers given fire and harvest.

Suppose P_c is the social value of a ton of sequestered carbon. Given harvest age a and absorption rate $\dot{C}(a)$, the value of carbon absorbed by a hectare stand, compounded to the harvest date a, is $P_c e^{ra}\int_0^a e^{-rz}\dot{C}(z)dz.$ At harvest, a proportion d of this carbon is quickly transferred back to the atmosphere through products such as chips, sawdust, paper and fuelwood that are either burned or quickly decompose. The remainder of the carbon goes into long-term product storage. If this decomposes at rate δ, the total value of the lost carbon as of date a is $[d + \delta(1-d)/(r+\delta)]P_c C(a)$, where C(a) is the carbon stock in the trees, so the net sequestered carbon value given harvest is:

$$(11) \qquad C_h = P_c e^{ra}\int_0^a e^{-rz}\dot{C}(z)dz - P_c C(a)\left[\frac{rd+\delta}{r+\delta}\right].$$

If the stand burns at time x < a, all the carbon is released, so the corresponding C_b is equation (11) with d = 1 and $\delta = 0$. Weighting C_b and C_h by the burn rate, λ, the overall expected value of sequestered carbon for a single rotation is:

$$(12) \qquad \begin{aligned} VC(a,\lambda) = {} & P_c\left(\int_0^a \lambda e^{-\lambda x}\int_0^x e^{-rz}\dot{C}(z)dzdx - \int_0^a \lambda e^{-(r+\lambda)x}C(a)dx\right) \\ & + P_c e^{-(r+\lambda)a}\left(e^{ra}\int_0^a e^{-rz}\dot{C}(z)dz - \frac{rd+\delta}{r+\delta}C(a)\right). \end{aligned}$$

To calculate this value for Douglas fir, we need values for the additional parameters P_c, d, and δ. We shall employ the carbon value calculated by Nordhaus(1993), who estimated a shadow price of carbon to promote climate stabilization of $P_c =$ \$100 per ton. Harmon, Ferrell and Franklin (1990) also estimated that approximately 40 percent of merchantable timber (net of breakage and defects) goes into long term storage, so d=.4. If this wood decomposes at 1%, then $\delta = .004$ is appropriate. Given these parameter values, Figure 5 shows the relationship of the expected sequestered carbon value with the burn rate, as well as with the planned harvest age. From the figure it is apparent that at the current rate of 8=.014 there is no planned rotation age maximizing the VC function, while at 8 = .028 it is maximized with a planned rotation age of approximately fifty years.[9] At a low fire probability it is better to allow the carbon to accumulate in living trees, while if the chance of a burn is high, then storage in wood products results in a lower expected rate of atmospheric transfer. The increased risk also significantly reduces the function's value at any harvest age. With a 50 year rotation, a hectare of forest sequesters over its lifetime an amount of carbon with an expected present value of $67,200 if 8=.014, while if the burn rate rises to 8 = .028 this expectation declines to $29,300 in present value.

From society's point of view, the planner should add VC, the expected carbon value, to the expected commercial value of the forest (V). With a given burn rate, the socially optimal planned rotation length over an infinite horizon is now the one maximizing (V(a,8) + VC(a,8)) /(1 - D_x(a,8)), where D_x is the expected rotation length. Compared with the commercial optimum of 43.8 years, at the current 1.4% burn rate it is never socially optimal to harvest the trees, while at 8 = .028, there is a socially optimal harvest at 47.1 years, still exceeding the 37.5 year commercially optimal rotation age given that burn rate. Thus, with a low risk of fire, it is optimal both from the commercial and social standpoint to store carbon in living trees, but this is no longer the case when the fire risk increases as the climate warms. With a high fire risk it is more prudent to store the carbon in forest products. Although the inclusion of carbon benefits means that society should rotate the forest less frequently than the commercial rent-maximizing rotation age,[10] both the commercially and socially optimal rotation lengths decline as the fire risk increases.

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[9] The derivative dVC/da is proportional to (rd + *) + (1-d)[dlnC(a) - 8], so it is always positive if the absorption rate dC(a)/C(a) exceeds the burn rate.

[10] This is confirmed by the fact that dVC/da is positive at the simulated rent-maximizing rotation ages of the previous section.

5. Conclusions

This paper has built a Faustmann-based model to investigate the effects of increased climate-induced fire and pest disturbance risk on the optimal rotation period in a commercial forest. Simulations using species of trees prevalent in North American forests indicate that as long as forestry is profitable both the commercial and social optimum rotation ages decline as the risk increases. This occurs despite the fact that the inclusion of carbon sequestration benefits in society's maximand means that the socially optimal rotation length exceeds the length that is commercially profitable.

The increased fire risk as the climate warms also has important implications for the ability of forests to act as absorbers of carbon. The arguments of the 'Umbrella Group' of countries who desire to use their forests carbon-absorbing ability to offset their need for fossil fuel emission reductions will have increasingly less force as the climate warms.[11] Because the heightened fire risk significantly reduces the ability of living forests to act as carbon sinks, alternative proposals for storing carbon by 'pickling' wood in cold lakes look increasingly attractive.

[11] Canada is a member of the Umbrella Group of countries. Until recently, Canada's forests created a net carbon sink, sequestering about 200 million tonnes of carbon per year, enough to offset her high anthropogenic emissions of 128 million tonnes in 1987 (Van Kooten *et al.*, 1992). However, the net ecosystem productivity of Canadian forests has declined in recent decades, actually creating net carbon emissions in the forest sector alone of approximately 75 million tonnes in 1990 (Kurtz and Apps (1995). The total Canadian net flux of approximately 200 million m.t. is nearly 7 per cent of the estimated annual world atmospheric carbon increase of 2,900 million m.t.

Table I: Growth Functions for Selected Tree Species
Douglas Fir, Northern Jack Pine, and White Spruce

Species	Growth Equation	Age of Maximum MAI (v/a)
Douglas fir	$v(a) = \exp(6.874 - 75.744/a)$	75.7
Jack pine	$v(a) = 397.49 - 1865.5a^{-.5}$	49.6
White spruce	$v(a) = 483\ [\exp(-.4181 \ln(a) + .15529 \ln(a)^2 - .0088 \ln(a)^3) - 1]$	130.0

Table II: Calibration Simulations for Selected Tree Species with α=0 (no warming)

Simulation	Douglas Fir	Jack Pine	White Spruce
Faustmann case: no fire risk (λ=0) and r=.02	51.4	41.0	58.3
Constant fire risk (λ= .014; α =0)	43.8	37.5	49.5
Constant fire risk with r = .034	43.8	37.5	49.5
Constant fire risk with r=.02, λ= .028, α=0	39.1	35.2	44.7

Simulations assume r =.02, and c/p = 17.49, 7, and .1, for Douglas fir, jack pine and white spruce respectively.

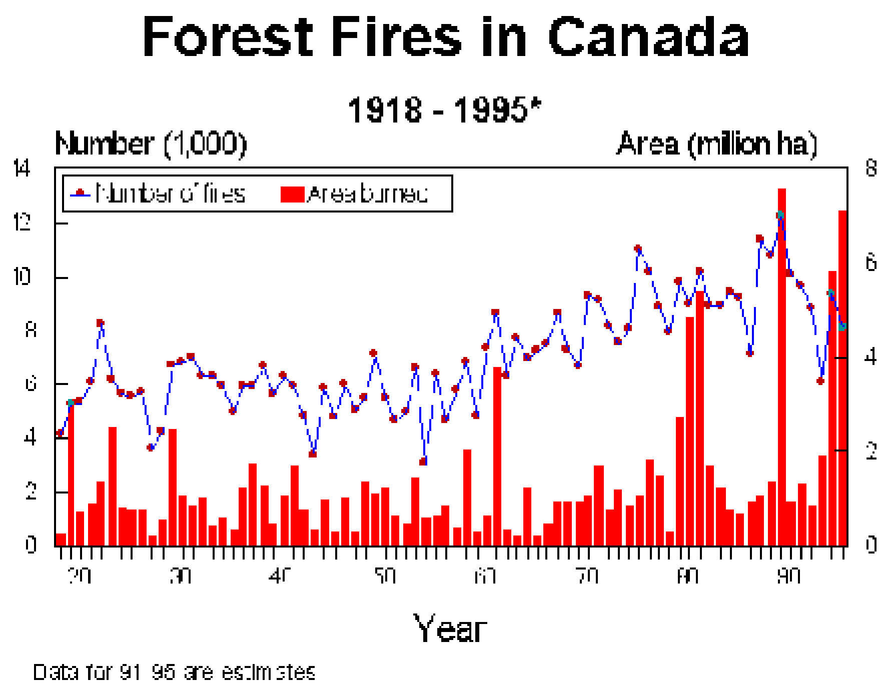

Figure 1: Forest fire statistics from the Canadian National Forest Service

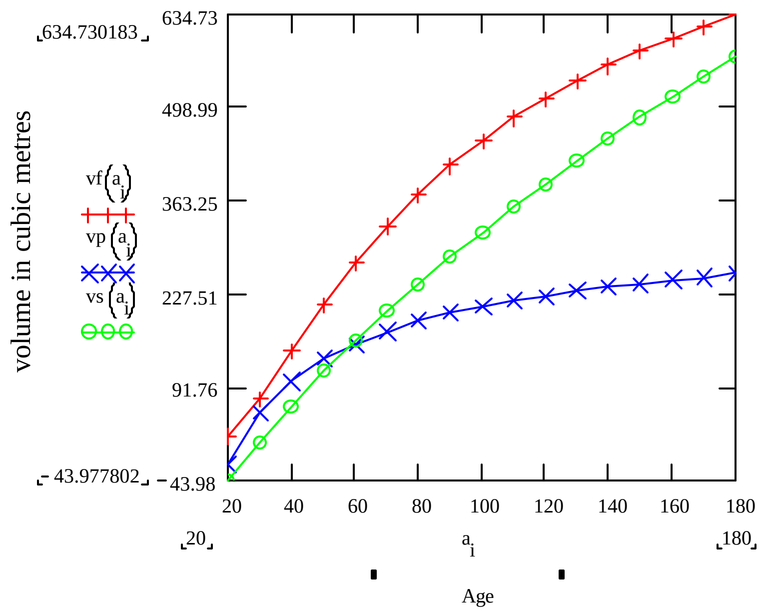

Figure 2: **Volume Growth Curves for Douglas Fir, $v_f(a)$, Jack Pine, $v_p(a)$, and White Spruce, $v_s(a)$, in m^3 per Hectare.**

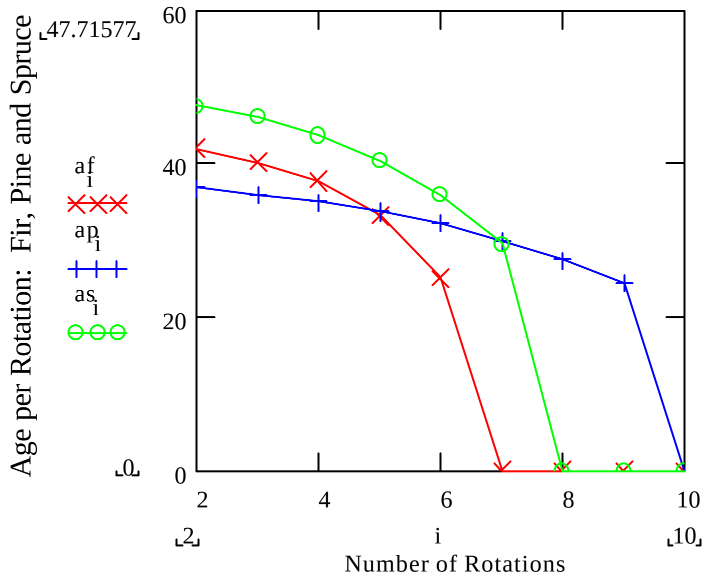

Figure 3 **Simulated Sequence of Rotation Ages for Douglas Fir, Jack Pine, and White Spruce with Increasing Fire Risk**

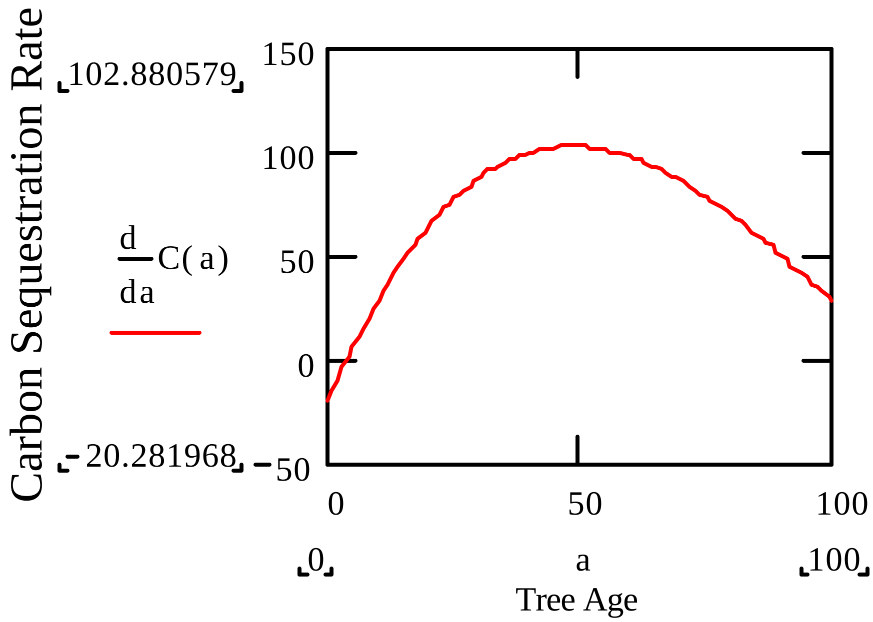

Figure 4 **Annual Carbon Absorbed by a Hectare Stand of Douglas Fir of Age a (tons/year)**.

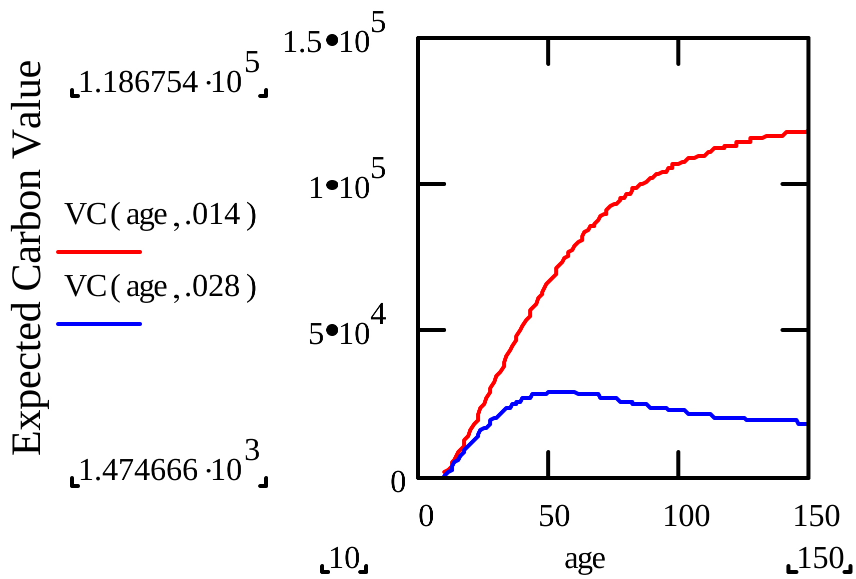

Figure 5 **Expected Present Value of Carbon Sequestered in a Hectare Stand of Douglas Fir, by Age of Stand, with 8= .014 and 8=.028.**

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